AP CALCULUS BC	YouTube Live Virtual Lessons	Mr. Bryan Passwater Mr. Anthony Record
Topic: Unit 10*	**Convergence and Taylor Polynomials** **Free Response Question Review**	**Date:** April 13, 2020

* The Topics in this lesson will only be those that will be directly tested on the 2020 AP Calculus BC Exam

Topic Name	Topic #	Quick Synopsis
Working with **Geometric Series**	10.2	**CONVERGENCE OF A GEOMETRIC SERIES** **1.** If $\lvert r\rvert<1$, the geometric series $\sum_{n=0}^{\infty} ar^n$ converges **2.** If $\lvert r\rvert\geq 1$, the geometric series $\sum_{n=0}^{\infty} ar^n$ diverges. **SUM OF AN INFINITE GEOMETRIC SERIES** If $\lvert r\rvert<1$, the geometric series $\sum_{n=0}^{\infty} ar^n$ converges, and its sum is $S=\frac{a}{1-r}$, where a is the first term of the geometric series and r is the common ratio.
Harmonic and *p*-Series	10.5	**CONVERGENCE OF A *p*-SERIES** The p-series is defined by the following where p is a positive real number. $\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots+\frac{1}{n^p}+\cdots$ **1.** converges if $p>1$, and **2.** diverges if $0<p\leq 1$.
Alternating Series Test for Convergence	10.7	**ALTERNATING SERIES TEST** Let $a_n>0$. The alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ and $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ converge if the following conditions are both met: 1. $\lim_{n\to\infty} a_n=0$ 2. $a_{n+1}\leq a_n$ for all $n>N$ where N is an integer
Ratio Test for Convergence	10.8	**THE RATIO TEST** Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero terms. **1.** Let $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert=L$, a number. • If $L<1$, then the series $\sum_{n=1}^{\infty} a_n$ converges absolutely. • If $L=1$, then the ratio test provides no conclusive information about the convergence or divergence of $\sum_{n=1}^{\infty} a_n$. • If $L>1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges. **2.** Let $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert\Rightarrow\infty$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
Finding **Taylor Polynomial Approximations** of Functions	10.11	**DEFINITIONS OF *n*TH TAYLOR POLYNOMIAL AND *n*TH MACLAURIN POLYNOMIAL** If f has n derivatives at c, then the polynomial $P_n(x)=f(c)+f'(c)(x-c)+\frac{f''(c)(x-c)^2}{2!}+\cdots+\frac{f^{(n)}(c)(x-c)^n}{n!}$ is called the nth **Taylor polynomial** for f at c. If $c=0$: $P_n(x)=f(0)+f'(0)(x)+\frac{f''(0)(x)^2}{2!}+\cdots+\frac{f^{(n)}(0)(x)^n}{n!}$ is called the nth **Maclaurin polynomial** for f.

2020 FRQ Practice Problem BC1

BC1 Let $a_n = \dfrac{(-1)^n}{n^{p-2}}$ and $b_n = \dfrac{-2}{n^{6-p}}$

(a) Let $p = 2.5$. Show that both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge.

(b) Find all integer values of p such that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge.

(c) Let $p = 4$. Let $f(x)$ be a function with derivatives of all orders at $x = 2$ with $f(2) = -3$ and where $f^{(n)}(2) = n! \cdot a_n$ for $n \geq 1$. Find $P_3(x)$, the third degree Taylor polynomial for $f(x)$ centered at $x = 2$.

(d) Using $P_3(x)$ that you found in part (c), find $P_3'(x)$. When $x = 3$, the series $\sum_{n=1}^{\infty} c_n$ is a p-series whose first three terms correspond to the three terms of $P_3'(x)$. Determine whether $\sum_{n=1}^{\infty} c_n$ converges or diverges when $x = 3$.

2020 FRQ Practice Problem BC2

BC2 Consider the series $\sum_{n=0}^{\infty} a_n$ where $a_n = \frac{5(x+3)^n}{(-6)^n}$.

(a) Determine if $\sum_{n=0}^{\infty} a_n$ converges or diverges when $x = 1$.

(b) Let $\sum_{n=0}^{\infty} a_n = L$ where L is a real number. Show that there is a value of x such that $L = 15$.

(c) Let $d_n = \frac{a_n}{n+1}$. Find the interval of convergence for $\sum_{n=0}^{\infty} d_n$.

(d) Let $f(x)$ be a function that is twice differentiable at all x values. If the first three terms of $\sum_{n=0}^{\infty} d_n$ are the second degree Taylor polynomial for $f(x)$ centered at $x = -3$, find $f''(-3)$.

2020 FRQ Practice Problem BC3

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
1	-2	0	3	4
4	$-\frac{9}{4}$	$\frac{3}{2}$	$-\frac{9}{4}$	$\frac{9}{2}$

BC3 The functions f and g are differentiable for all orders at all x values. Selected values for f and several of its derivatives are given in the table above. The function g is defined by:

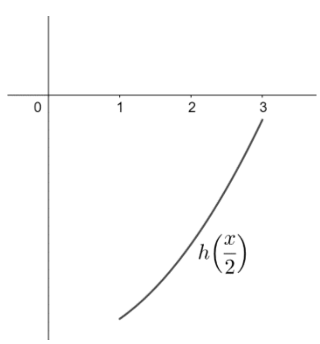
$$g(x) = 3x + \int_4^{4x} f(t)\,dt$$

(a) Find $P_3(x)$, the third degree Taylor polynomial for $f(x)$ centered at $x = 1$.

(b) Find $T_3(x)$, the third degree Taylor polynomial for $g(x)$ centered at $x = 1$.

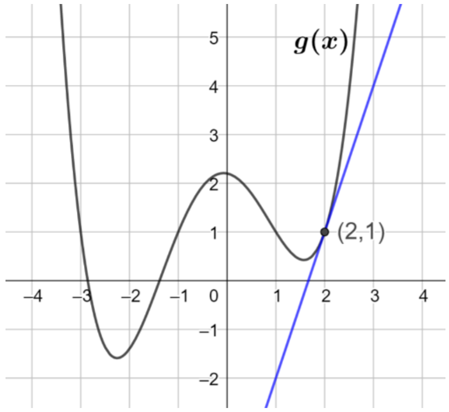
(c) Let $\sum_{n=0}^{\infty} a_n$ be a geometric series whose first four terms are the four terms of $T_3(x)$ found in part (b).

Find $\sum_{n=0}^{\infty} a_n$ where $x = \frac{5}{4}$ or show that the series diverges.



(d) A portion of the function $h\left(\frac{x}{2}\right)$ is above. Explain why $h\left(\frac{x}{2}\right)$ could not be the graph of $f(x)$.

2020 FRQ Practice Problem BC4



BC4 A function g has derivatives of all orders for all values of x. A portion of the graph of g is shown above with the line tangent to the graph of f at $x = 2$.

Let h be the function defined by $h(x) = x - 2 - \int_2^{2x} g(t)\,dt$.

(a) Find the second degree Taylor polynomial $T_2(x)$, for $h(x)$ centered at $x = 1$.

(b) Explain why $P_2(x) = 1 + 3(x-2) - \dfrac{5(x-2)^2}{2!}$ could not be the second degree Taylor polynomial for $g(x)$ centered at $x = 2$.

(c) Consider the geometric series $\sum_{n=0}^{\infty} \dfrac{a_n}{(2n)!}$ where the first three terms of a_n correspond to the three terms for $T_2(x)$. Find $\sum_{n=0}^{\infty} \dfrac{a_n}{(2n)!}$ when $x = 0$.